The Pattern of Knowledge Flows between Technology Fields: modularity, near-decomposability and autocatalytic sets.

(Preliminary and incomplete draft)

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1 Knowledge pattern and innovations

We think of the disciplinary technological knowledge available in the economy as subdivided into different disciplinary fields, or simply fields, which only partly correspond to application domains. A field identifies a set of funcional or '*phenotipic*' traits in the broad technology domain. The set of technology fields existing at a given date and the relations between them are understood to be the outcome of the way in which the human society has historically explored and exploited the set of possibilities offered by the physical world. Technological determinism is therefore inappropriate, in that there is a social element inherent to the architecture of technology. Innovations, that is, additions to the stock of technological knowledge, can be incremental, radical, or network. The scope of a disciplinary-field definition is sufficiently narrow that radical innovations are understood as exogenous events affecting the number and quality of technology fields. Network innovations are changes in the matrix describing the active cross-field learning interfaces and the strength of these knowledge connections. Incremental innovations are those additions to the knowledge stock originating in a specific field that do not affect the set of available fields or the design of cross field interactions.

The average number of incremental innovations per unit of time in a given field depends on two main factors: in the first place, the set of innovation opportunities available in that field, in the second place the innovation effort in the same field. In this paper we assume that innovation opportunities are primarily determined by the progressive local knowledge base. This consists of the subset of ideas that are known by R&D laboratories currently operating in the given technology field and that are potentially conducive to useful recombinations and developments leading to new disciplinary knowledge. Under a recombinant interpretation of knowledge growth (Reiter [27], Weitzman [35]), the progressive knowledge base can be regarded as the repertoire of recombination possibilities from which innovations will originate. So defined, the progressive local knowledge base partly consists of ideas originated from past innovations in the same technology field, but will also partly consist of ideas originated from past innovations in other fields and that are made available to the field in question by the knowledge interfaces that are currently active across technologies. The knowledge pattern is the set of knowledge interfaces that are active across fields, together with their degree of activation. More precisely, the intensity c_{ij} of knowledge transfer from field j to field i is the average frequency with which an innovation in field j gives rise to ideas that are relevant to innovation activity in field i. This implies that c_{ij} will not fully capture the occasional transfer of a radically new idea from j to i, unless we have a reliable way of weighing the importance of ideas. The absence of such reliable weights is made less dramatic by the fact that a radical innovation is normally followed by a swarm of incremental innovations, so that the frequency c_{ij} of systematic knowledge transfer will at least partly reflect the importance of ideas.

The way in which the structure of the knowledge pattern evolves through time is shaped by radical and network innovations. This paper proceeds on the bold hypothesis that the organization of a knowledge pattern in a given historical period, say the last decades of the 20th century, reflects not only the key technological interfaces that are dominant in the period, for instance those concerning the information and communication technologies (ICT). We expect that the organization of these interfaces will partly reflect more general principles, bearing upon the way in which the accumulation of new ideas over time affects the complexity of innovation activity.

To clarify this point, it is best to think of an idea as a specific configuration of a set of basic codifiable knowledge components. Ideas discovered in different fields may share some of their basic components and for this reason R&D in one field may be relevant to R&D in others. Our premise is that exploiting the 'relevance' of a knowledge input to the discovery of a knowledge output requires that the configuration of the latter conforms to a number of constraints imposed by the configuration of the former. The reason is that the relative fitness of ideas in performance space is strongly affected by relations of interdependence or complementarity and this makes the problem of finding the best configuration of a *given* set of knowledge components difficult. In other words, technological fitness landscapes are rugged [16]. In the landscape metaphor, the output of incremental R&D is an expansion in the known surface of the given landscape At a given date, the dimension of the landscape depends on the number of basic knowledge components available, which we assume a strictly increasing function of the number of known fields. The dimension of disciplinary R&D landscapes tends to increase through time together with the number of fields, as a result of radical innovations.

The ruggedness of the fitness landscape facing a R&D laboratory operating in a given field i is produced by the complementarities between the components which define the knowledge space of field i.(For the sake of simplicity, we assume that R&D laboratories operating in the same field face the same knowledge space). On such 'complicated landscapes boundedly rational R&D actors have strong incentives to adopt local search euristics, which are able to climb only local optima. The waiting time to attain globally optimal solutions grows exponentially with landscape dimension, so that global serach euristics will not pay in the arena of competition. The ruggedness of the landscape facing R&D in field i is determined, for a given dimension, by the interdependencies that are specific to the technology field in question.

Ruggedness does not fully define the complicatedness of R&D in one field. The reason is that, the *shape* of the fitness landscape and possibly its dimension, changes through time as a result of deformations induced by discoveries in R&D laboratories operating in other fields (there is strategic interdependence between R&D choices). In other words, R&D landscapes of different disciplinary fields are more or less tightly coupled[1]. Taken together, ruggedness and degree of coupling are the sources of the '*complicatedness*' facing R&D activity.

Evolvability in the technological knowledge domain requires, much like in other domains, that the complicatedness of interactions does not grow in propotion with the inevitable growth in the scale of the system (as measured for instance by the number of technology fields). As we shall see, there is convincing evidence that this is achieved through a selection for modularity in the organization of the learning interfaces between technology fields. Since the notion of modularity has recently acquired a variety of meanings in the literature, it is worth spelling out our use of the term: it is possible to partition the set of technology fields into subsets called modules, such that on average, the intensity of the knowledge links a field sends to or receives from the fields participating in the same module is higher than the corresponding average intensity of the links between a field in the module and a field belonging in a different module. The lower *average* intensity of between module links is not necessarily consistent with the stronger requirement that every such link is 'weak', so that between module links are negligible over appropriate time scales (Simon's [31] near-decomposability). It is however consistent with the notion that between module communication takes place in aggregate form (hierarchic aggregability). Hierarchic aggregability in the organization of knowledge spaces is sufficient in order that the waiting time to find a globally optimal solution on a landscape is a polynomial rather than exponential function of the landscape dimension ([36]). As system size increases, hierarchic modular structures develop, but the knowledge pattern so originated may not exhibit the more demanding organizational structure of a near decomposable system a la Simon.([33] , [31] and [32]). As will be argued, the near-decomposability condition that the size of every inter-module relation is 'weak' (has a lower order of magnitude), compared to

[1] S. Page [23] associates '*difficulty*' of search with the fact that fitness lanscapes are rugged, so that difficulty increases with ruggedness, and associates '*complexity*' of search with the fact that fitness landscapes are coupled in a way that a search step in one induces a deformation in the others. In Page's definition, complexity is the a measure of how tight is the coupling between the landscapes. In the sequel we shall not exploit Page's distinction.

the size of every intra-module relation, appears to be violated by the structure of an empirical knowledge pattern. We conjecture that the modular architectures of a knowledge pattern are better described by the conditions of hierarchic aggregabilty that give rise to the design rules of compositional evolution [36], and that the tendency of knowledge-pattern complicatedness to increase with scale is kept under control through the development of flexible modular architectures of this type.

Still, Simon's idea of identifying functional or structural subset of a network, by separating first-order from lower-order magnitude links proves to be euristically insightful in the analysis of an empirical knowledge pattern. As will be shown, proceeding in this way enebles the identification of functional and structural units that define the strongest systematic and self sustaining mechanisms of knowledge transfer and accumulation within the network. These 'core' structures are defined by the connectivity property that every node (technology field) in the core is connected to every other node in the same core by a circular self-sustaining information flow. The core structure achieving the highest rate of knowledge transfer is dominant. In the approximation based on first-order magnitude links the dominant core structure will be typically a strict subset of the network. Other, non dominant cores will also coexist with it. We expect that in an empirical knowledge pattern the dominant core identified through 1st order size connections corresponds to the functional module of technology fields and knowledge interfaces that together identify the dominant technology paradigm of the period. As will turn out, the prediction is fully corroborated in the empirical analysis to follow.

The relevance of this approach in the analysis of knowledge transfer between technology fields is further motivated in a companion paper [3] which offers a theoretical model of the way a given knowledge pattern affects the distribution of incremantal innovations. In that paper, the qualitative model predictions were then matched with facts, showing that changes in the structure of the empirical knowledge pattern obtainable from patent-citation data could provide a clue to explaining changes in the empirical distribution of innovations. The architecture of the empirical knowledge pattern per se was largely unexplored, partly as a result of the relatively high level of aggregation at which the empirical analysis was carried out (technology fields were identified with two digits technological subcategories as defined in [11]). This paper extends the analysis of [3] in two directions. In the first place, the architecture of the empirical knowledge pattern based on USPTO patent citation data is investigated at the much finer level of resolution of 3-digits technology classes. Based on this fine grained analysis, the issues concerning the relevant notion of modularity, the core structures and their functionality with respect to patent distribution, can be addressed more rigorously. In the second place, the changes through time in the cross-field architecture of knowledge transfer are investigated at the same finer level of resolution. The envisaged structural change in the period 1975-1999 offers a few guide lines of interpretation consistent with the idea that the information and communication technologies (ICT), although representing the core of knowledge creation throughout the period, only in the second half became fully integrated with the other sectors. The suggested guidelines are broadly consistent with general ideas on structural change suggested in the evolutionary applied and theoretical literature on knowledge creation.

The paper is organized as follows. In the next section we introduce our formal description of a knowlegde pattern and the precise notions of modularity, near-decomposability, and core structures that will be used in the rest of the paper. Section 3 relates our reconstruction of an empirical knowledge pattern from the NBER files of patent-citation data to the growing literature on knowledge spillovers based on patent citations. Section 4 exploits the notions of modularity and ACS to analyze the architecture of the re-constructed empirical knowledge pattern in the periods 1975-1986 and 1987-1999. Section 5 concludes.

2 Knowledge pattern, modularity and autocatalytic sets

2.1 The connection matrix C

We consider an economy with a finite set $S = \{1, ..., n\}$ of known technology fields. A field j is here understood as a (possibly infinite) set T_j of potential configurations, or designs. The technological state of the economy is defined by $\{G(S, L, C), A\}$. A_i $i = 1, ..., n$, is the number of useful ideas cumulatively produced by R&D in field i. $G(S, L, C)$ is a weighted directed graph, with a set S of nodes, that are here interpreted as technology fields, a set L of directed knowledge links between these nodes, and a connection matrix C of weights, or intensity coefficients, attached to the links in question. c_{ij} is the strength of the directed link from j to i. It is a measure of the extent to which ideas developed in sector j are relevant to R&D in sector i, in the sense that A_j expands the knowledge base of the latter.[2] We can safely assume that some of the knowledge produced by past innovations in one field is always relevant to R&D activity in the same field, that is, $c_{ii} > 0$, $i = 1, ..., n$. By definition, C satisfies the condition: $c_{ij} = 0$ if and only if the directed link $(j \to i) \notin L$. This justifies the definition:

Definition 1 $G(S, L)$ *is the unweighted directed graph associated with the weighted directed graph* $G(S, L, C)$*, or, more sinthetically, with* C*.*

The discovery which brings j in the set S of known technologies, brings also the knowledge stock A_j to its lower bound $A_j = 1$; after that, A_j grows as a result of the cumulative flow of incremental-innovation arrivals in the technology field j. Let $a_i = A_i / \sum_j A_j$. [3] builds a dynamics of the column vector a of share distributions $a_i, i = 1, ..., n$, driven by the flows of knowledge inputs across fields. The flow of useful innovations in sector i depends on two factors, the effective

[2] In our interpretation, $c_{ij} = 1$ if every idea developed in field j is a relevant knowledge input to R&D activity aimed at developing a new idea in field i. Since ideas are non rival, it may well be the case that $c_{ij} > 1$.

R&D effort in this field, Q_i/A_i, and the repertoire of available ideas that are the 'building blocks' of R&D in field i; this repertoire corresponds to the knowledge flows $\sum_j c_{ij}A_j$ received by i through the active interfaces described by C. The stock A_i, $j = 1, ...n$, evolves according to the differential equation:

$$\dot{A}_i = \sigma \frac{Q_i}{A_i} \sum\nolimits_j c_{ij}A_j \tag{1}$$

where σ is a uniform productivity[3] parameter. It can be redily verified that in the long-run condition such that the effective R&D effort Q_i/A_i is uniform across fields, every right eigenvector of C is a dynamic equilibrium of the differential equation above. Under the hypothesis that relative R&D effort in field i, $Q_i/\sum_i Q_i$, increases (decreases) depending on the extent in which innovation opportunities in this field, $\sum_j c_{ij}A_j$ is higher (lower) than average, it is proved[4] that the dynamics of a converges to a fixed point a^* which is the right eigenvector of C associated with the Perron-Frobenius eigenvalue λ^*. (Using a genericity argument, λ^* is assumed to have multiplicity 1). The suggested interpretation is that λ^* is the highest long-term sustainable rate at which the connection matrix C makes knowledge inputs avilable to the technology fields which enter the non negative (right) eigenvector of C associated to λ^*. The suggested interpretation is that the positive entries in this eigenvector define the fields participating in the dominant technology paradigm. For the sake of completeness, the model dynamics is reported in appendix A.

2.2 Modularity of C

In our matrix C, the degree of activation c_{ij} of the knowledge transfer from field j to field i is a measure of the probability that an idea discovered in field j is *relevant* to the discovery of a new idea in field i, if exploited by R&D in this field. We may note, in passing, that the same idea discovered in one field, may be relevant to many other fields; hence, there is no implication that the elements in the columns of C add up to 1. For the sake of simplicity, we think of relevance as a binary variable which may take value 0, *not relevant*, or 1, *relevant*. Under the above interpretation, the exploitation by field i of a relevant input idea discovered in field j requires meeting the constraints carried by such input. A relatively high (low) value of c_{ij} is also an indication that the landscape of field i is tightly (weakly) coupled to that of field j; the present directions of useful discovery in the former are strongly (weakly) conditioned by the direction in which the configuration of useful ideas has been developing in the latter.

Mutually high values of c_{ij} and c_{ji} signal a coevolution of the directions of discovery in fields i and j. The constraints imposed by such a coevolution bring

[3] The simplifying assumption follows from the fact that there is no attempt at weighing the quality of a knowledge transfer, but at measuring its frequency together with its source and destination.

[4] The proof is given in [3] for low dimensional n. By way of simulations, the result is conjectured to hold for any given n.

in a trade-off. The price to pay for the oppotunity of a faster progress through the formation of compatible knowledge standards in fields i and j, and the opening of active interfaces between them, is a reduced capability to move away from the direction specified by those standards. This creates the danger of a technological lock in, because trajectories traced by local search procedures are path dependent. Moreover, if global search criteria should occasionally benefit from a lucky stroke and envisage new and potentially useful search directions, R&D in these directions can not avail itself of the vast array of knowledge inputs and complementary ideas that are maide available by the cross-field interfaces corresponding to the prevailing knowledge pattern. For this reason, the idea occasionally discovered through a first research step in a radically new direction is likely to perform poorly as compared to the best-practice disciplinary knowledge. The transition to a different dominant pattern may prove difficult.

It is worth stressing that under our interpretation, the search 'complicatedness' faced by R&D in field i is not linearly additive in the parameters c_{ij} composing the ith row of C . The reason is that the constraints imposed by the exploitation of knowledge spillovers from a given field h conform to the set of dominant designs prevailing in h. Exploitation of spillovers from a larger number of diverse fields requires compliance to a wider set of qualitatively different constraints. The complexity of the search space facing R&D in field i comes to depend not only on the total sum $\sum_j c_{ij}$, but, more importantly, on the distribution$\left[\frac{c_{ij}}{\sum_j c_{ij}}, i=1,...,n\right]$ and on the technological diversity between the fields from which field i draws its knowledge inputs. Ceteris paribus, R&D receiving its knowledge inputs from a smaller number of qualitatively more similar technology fields is expected to face a less complicated search space.

The observation above identifies a strong incentive for field i to concentrate the incoming knowledge links of total intensity $\sum_j c_{ij}$ across a restricted number of technologically similar source fields. In other words, we expect a selection for modularity in the structure of C. Intuitively, the set of n fields can be partioned into $m < n$ disjoint groups, such that , *on average*, and in ways that will be specified below, the within group links are stronger than the between group links. It is also worth observing that the argument can be replicated at different hierachic levels; but to the extent that there is qualitative variation in the nature of technological constraints, activities and functions at different levels of the hierarchy, there is no direct implication that the organization of knowledge patterns is necessarily self-similar across modules and at every scale of resolution.

2.3 Modularity and dynamics: near-decomposition, aggregation and core structures

The above intuitive and quite genaral idea of modularity of the connection matrix C admits a quantitative expression, based on recent contributions in network theory and applications. Suppose that the set $N = \{1,...,n\}$ of technology fields is partitioned into m disjoint subsets, or groups, so that $N =$

$N_1 \bigcup N_2 \bigcup ... \bigcup N_m$, where $N_h$is the set of fields belonging to group h. The total intensity of an outword link from group h directed to itself or to other grpups is $\hat{a}_h = \sum_i \sum_j c_{ij}, j \in N_h, i = 1, ..., n$. The corresponding total intensity of an inword link to gruop h from itself or from other groups is $\check{a}_h = \sum_j \sum_i c_{ij}, i \in N_h, j = 1, ..., n$. If the total intensity of links in C is $T = \sum_i \sum_j c_{ij}, i, j = 1, ..., n$, then the average relative frequency with which an outword link in C originates from, and arrives to, group h is $\hat{e}_h = \frac{\hat{a}_h}{T}$ and $\check{e}_h = \frac{\check{a}_h}{T}$, respectively. The modularity measure Q_h of the links from and to gruop h in the context of the given network C, is then expressed by the extent in which the frequency of within-group links exceeds the frequency which would be expected from the hypothesis of a random wiring.

$$Q_h = \left[\sum_{i \in N_h} \sum_{j \in N_h} c_{ij} \right] - \hat{e}_h \check{e}_h \tag{2}$$

The modularity of C according to the partition $\{N_1, ..., N_m\}$is then expressed by the sum $Q = \sum_h Q_h$and the appropriateness of two alternative partitions of N is evaluated by choosing the partition yielding a higher value of Q. In this spirit, the modularity of C is defined by selecting the Q-maximizing partition ([21]). Since the Q modularity of the null partition $\{N\}$ is zero, the Q modularity of C takes values in the interval $[0, 1]$.

The situation is illustrated by the following example of a connection matrix showing the mutual links between $n = 8$ fields. Symbols c of different size represent links of different order of magnitude. The set of fields $\{1, ..., 8\}$ can be partitioned into two modules $N_a = \{1, ..., 4\}$ and $N_b = \{5, ..., 8\}$, such that the strength of the links between N_a and N_b is at least one order of magnitude.lower than the strength of the within module links. Each module N_a and N_b can be further partitioned into two sub-modules N_{a1}, N_{a2} and N_{b1}, N_{b2}, with the same property that the strength of the links between sub-modules is at least one order of magnitude lower than the strength of the links within sub-modules.

$$\begin{bmatrix} C & C & c & c & {\scriptstyle c} & {\scriptstyle c} & {\scriptstyle c} & {\scriptstyle c} \\ C & C & c & c & {\scriptstyle c} & {\scriptstyle c} & {\scriptstyle c} & {\scriptstyle c} \\ c & c & C & C & {\scriptstyle c} & {\scriptstyle c} & {\scriptstyle c} & {\scriptstyle c} \\ c & c & C & C & {\scriptstyle c} & {\scriptstyle c} & {\scriptstyle c} & {\scriptstyle c} \\ {\scriptstyle c} & {\scriptstyle c} & {\scriptstyle c} & {\scriptstyle c} & C & C & c & c \\ {\scriptstyle c} & {\scriptstyle c} & {\scriptstyle c} & {\scriptstyle c} & C & C & c & c \\ {\scriptstyle c} & {\scriptstyle c} & {\scriptstyle c} & {\scriptstyle c} & c & c & C & C \\ {\scriptstyle c} & {\scriptstyle c} & {\scriptstyle c} & {\scriptstyle c} & c & c & C & C \end{bmatrix}$$

The Q-modularity of the above connection matrix C is clearly maximized by the partition $\{N_{a1}, N_{a2}, N_{b1}, N_{b2}\}$ of N. Long ago Simon and Ando observed that if a square non negative matrix like C in the example describes the equations of motion of a (locally) linear dinamical system of n variables $(X_1, ..., X_n)$

of the form[5]

$$\dot{X}_t = CX_t \tag{3}$$

and if the ratio

$$\frac{c}{C}$$

is sufficiently close to zero, the linear operator C is decomposable into the form

$$C = C^* + \varepsilon D$$

where C^* is block diagonal, with the diagonal operators $C^*_{a1}, C^*_{a2}, C^*_{b1}, C^*_{b2}$ acting on the components of X_t corresponding to the partition $\{N_{a1}, N_{a2}, N_{b1}, N_{b2}\}$, respectively. Simon and Ando provided the conditions for which the time scale of the dinamical system is approximately decomposable into a 'short-run' $t < T_0$, a 'medium run' $T_0 < t < T_1$, and a 'long run' $t > T_1$. During the short run, the dynamical behavior is dominated by the diagonal block operators acting on the relevant components of x_t, that is, it is almost completely determined by the within partition relations. During this interval the dinamical system is nearly decomposable, but is not aggregable, because the within-partition components of X_t, namely $X_{a1,t}, X_{a2,t}, X_{b1,t}, X_{b2,t}$ have not yet completed their convergence to the dominant eigenvectors $a^*_{a1}, a^*_{a2}, a^*_{b1}, a^*_{b2}$ of the diagonal block operators. This convergence marks the inception of the medium run. During this interval the within partition dynamics approximated by C^* still dominates, so the system is still decomposable, but to the extent that the within partition distributions are closely approximated by $a^*_{a1}, a^*_{a2}, a^*_{b1}, a^*_{b2}$, the system is also aggregable. In the long run, the between partition relations become relevant and for this reason C^* does not offer a good approximation of the dynamics any longer. During this interval the changes in X_t induced by the between partition relations are weighted by the equilibria of the within-partition distributions. For this reason the system is still aggregable, even though it is no longer decomposable.

The argument above shows that the conditions for the decomposition and aggregation of variables acted upon by a linear operator do not in general overlap, and have to be clearly distinguished. To clarify this point, which plays an important role in the sequel, it is worth considering an example concerning the dynamics induced by 3 on the share distribution variables:

$$x_{it} = \frac{X_{it}}{\sum_{i=1}^{n} X_{it}}$$

$$\dot{x}_i = \sum_{j=1}^{n} c_{ij} x_j - x_i \sum_{j,k=1}^{n} c_{kj} x_j \tag{4}$$

In this example the operator C lends itself to a form of aggregation, even though the conditions for decomposability fail.

$$C = C^* + \varepsilon D = \begin{bmatrix} c^*_{11} & c^*_{12} & 0 & 0 \\ c^*_{21} & 0 & 0 & c^*_{24} \\ c^*_{31} & 0 & c^*_{33} & 0 \\ 0 & 0 & 0 & c^*_{44} \end{bmatrix} + \varepsilon D$$

[5] [33] refers to the corresponding 1st order difference equation.

Here D and C^* are $n \times n$ non negative matrices, $n = 4$ and ε is 'sufficiently small'.The short run dynamics of the relative share distributions of $(x_{1,t}, .., x_{4,t})$ converges to the share distribution of the right eigenvector of C^* associated to its dominant eigenvalue λ^*. the aggregation referred to above is induced by the dominant eigenvector properties of C^*. For the sake of later reference we introduce the following definitions and remark.

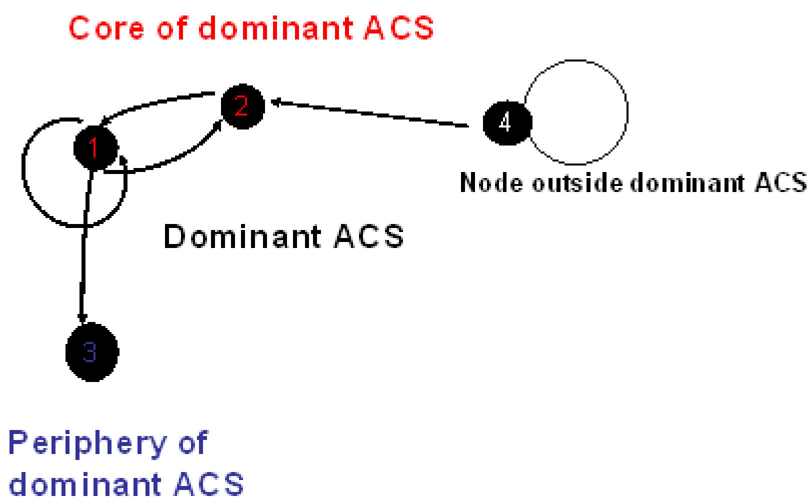
Definition 2 *For the graph $G(S, L)$ associated to a connection matrix C, a autocatalytic set (ACS) is a subgraph of $G(S, L)$ such that each vertex in the subgraph has at least one incoming link from some vertex of the subgraph (Jain and Krishna [14]). Notice that our assumption $c_{ii} > 0$, $i = 1, ..., n$, implies that $G(S, L)$ has n trivial ACSs. The* dominant ACS *of $G(S, L)$ is its largest subgraph with the property that the associated connection matrix C_a satisfies $\lambda^*(C) = \lambda^*(C_a)$.*

Definition 3 *For the dominant eigenvector a^* of the connection matrix C in the weighted directed graph $G(S, L, C)$, consider the subset $S_a \subseteq S$ of the vertices corresponding to the positive components of a^*, together with the subset $L_a \subseteq L$ of the links between them. $G(S_a, L_a)$ is the subgraph corresponding to a^* of the unweighted directed graph $G(S, L)$ associated to $G(S, L, C)$.*

Remark 4 *$G(S_a, L_a)$ is the dominant ACS of $G(S, L)$.*

With the tools above we can now look at the graph $G(S^*, L^*)$ induced by C^*. $G(S^*, L^*)$ is itself a ACS, because node 4 sends a link to itself, but provided that c_{44} is sufficiently small, node 4 does not belong to the dominant ACS of $G(S^*, L^*)$, which consists of the nodes 1, 2, 3 and the links between them. The reason is that node 4 does not receive links from the others; as a result, to the extent that c_{44} is strictly lower than the dominant eigenvalue of C^*, the fourth component in the dominant eigenvector of C^* is zero.

In the dominant ACS of $G(S^*, L^*)$, and in every ACS more generally, we distinguish a 'core' and a 'periphery'. The *core* subgraph of the ACS, which in our example is formed by vertices 1 and 2, and by the links between them, has the defining property that starting from any vertex of the core, any other vertex of the autocatalytic set can be reached following a sequence of directed links. This defining property of the core is labelled *closed path connectivity.* Vertices in the autocatalytic set that do not belong to its core, belong to its *periphery.* In our example the periphery of $G(S^*, L^*)$ consists of vertex 3, together with the link from vertex 3 to itself.



The form of aggregation enabled by the dominant eigenvector propereties of $C^*$is now revealed by the fact that the relative size (x_1/x_2) of the variables 1 and 2 in the core of the dominant ACS is independent of the measure of connections c_{ij} outside the core. In this sense, and in spite of the fact that the system is not decomposable (in Simon's sense), the variables x_1, x_2 exert an aggregate influence on the short-run convergence of variables outside the core. Moreover, the aggregation dominating the short-run equilibria will be also, if only approximately, felt in the long run, because ε is small.

The core of the dominant ACS of a knowledge pattern is the centre of the strongest self-sustaining mechanisms of knowledge creation and transmission within that pattern. In a relevant sense, the links connecting the core to the other fields in the dominant ACS disseminate building blocks ([13]) that are the aggregate outcome of the relations within the core. Mathematically, this corresponds ton the fact that the Perron-Frobenius eigenvalue of the dominant ACS is affected by any quantitative change of a connection coefficient within the core, independently of the wiring and intensity of the links from the core to the periphery. Euristically, this form of aggregation reflects the combinatorial view of knowledge creation adopted in this paper. A stronger link c_{ij} signals the increased capacity of the target field j to creatively recombine knowledge from the source field i with building blocks directly or indirectly received from other fields. Such building blocks are of course the outcome of previous creative recombinations. The closed path connectivity of the core is crucial, in this respect, in that it signals that the mechanisms in question are self sustaining. The Perron-Frobenius eigenvalue provides the aggregate measure of self-sustainingness.

The only reason to avoid the tighter coupling of the fields in a knowledge pattern through pervasive strong links is to avoid the corresponding growth in complexity carried by the need to set the technology standards. in one field in tune with those that are simultaneously evolving in other fields. The setting in tune will be easier, if the first order-size links that give rise to the closed path connectivity within C^* are relatively few in number. The point here is that the coordination between technology fields is more complicated if the relations between them are not strictly hierarchical (one -way), but contemplate a multiplicity of feed-back loops. such loops are characterisctic of the relations within the core, which are circular, with a multiplicity in the measure of closed path connectivity, which tends to grow with the number of first order links within C.

We conjecture that the relatively low dimension of the (first-order) dominant core facilitates the formation of well defined standards and smooth learning interfaces within the core. The incentives for a low-dimensional dominant core will be strongest during the early phase of design-standard formation within the core, because in this phase the process of knowledge creation is more turbulent ([1], [4]). The diffusion of the aggregate knowledge produced by the core to the periphery of the ACS is made less complex by the fact that the relations between the core and the periphery are hierarchical. To this extent, we expect that the ratio between the size of core and periphery is lower during the early turbulent phase of design standard formation.

3 Reconstructing knowledge spillovers from patent-citation data: a brief overview (missing)

4 The pattern of knowledge flows and innovation dynamics: 1975-1999

The data source for our exercise is the NBER Patent-Citations data file, as made available in Jaffe and Trajtenberg [15]. The main data set PAT63_99 contains all utility patents[6] granted by the U.S. Patent and Trademark Office (PTO) between January 1, 1963 and December 30, 1999. Among the variables that the PTO originally assigns to each patent, most relevant for us, in addition

[6] Utility patents constitute the overwhelming majority of patents, which include, in addition, design, reissue and plant patents. Cfr. Hall, Jaffe and Trajtenberg [11, p. 407, n. 4].

to the grant year, is the main U.S. patent class.[7] There were 417 patent classes in the classification in use in 1999. The 'original' variables assigned by the PTO to the various patents are enriched by the authors of the dataset with a number of 'constructed variables'. In particular, the 417 classes are aggregated by the authors into 36 technological subcategories and these further aggregated into 6 categories ('Chemical', 'Computers & Communications', 'Drugs & Medical', 'Electrical & Electronic', 'Mechanical', and 'Others'). The data set PAT63_99 can be profitably matched with a second data set, namely, CITE75_99, which contains all citations made to patents in PAT63_99 by patents issued between January 1, 1975 and December 30, 1999.

The first aim of our exercise is to obtain from the citations data just described, a computationally viable description of the knowledge flows between technology fields, and of the changes thereof. In the companion paper [3] the analysis was carried out resorting to a simplified description of technology fields according to their partition into 36 subcategories. This paper extends the analysis to the technological classification according to the 418 3-digit classes. To evaluate the intensity of knowledge spillovers across technology fields, we studied how far patenting in a class xy in a time interval $[t, t+z]$ was followed by citations to xy by patents issued in every other class in the time interval $[t+s, t+z]$. In this way, for each class xy, we obtained a 418-dimensional vector of citations to xy. The corresponding vector of spillover intensity from xy to the other classes was obtained by dividing the citations vector by the number of patents issued in xy in the period $[t, t+z]$. Proceeding in this way for each xy in the set of 418 classes, we arrived at a matrix of spillover intensity which is the empirical analogue of the matrix C in our model. To detect structural change, if any, in the pattern of knowledge spillovers in the period under study, we divided the latter into two sub-periods and obtained a corresponding analogue of matrix C for each sub-period.

The actual procedure followed was complicated by two types of considerations that have to do with those characteristics of the available data set, that are most relevant to our exercise.

The first relevant characteristic is that the number of citations in a finite time interval is affected by truncation effects related to backward and forward citation lags (Hall, Jaffe and Trajtenberg [11, pp. 421-424]). This imposed a choice of the subperiods in a way that comparisons between them were least affected by the unavoidable distortions introduced by truncation effects. In particular, the parameter s was held constant between the subperiods ($s = 12$) and differences in z were negligible ($z = 23$ in the first subperiod, $z = 24$, in the second). The corresponding choices for t were $t = 1963$ and $t = 1975$, respectively. For the sake of later reference, the intervals $[t+s, t+z] = [1975 - 1986]$ and

[7] The reason for the qualification 'main' is that each patent is assigned by the PTO to a 3-digit patent class and to a subclass, but also to any number of 'subsidiary' classes and subclasses that seem appropriate. Moreover, the system is continuously updated with new classes being added and others being reclassified or discarded. In this case, the PTO retroactively assigns patents to patent classes, according to the most recent clasification system. Cfr. Hall, Jaffe and Trajtenberg [11, p. 415.]

$[t + s, t + z] = [1987 - 1999]$ are referred to below as first window ($W1$) and second window ($W2$), respectively.

The second relevant characteristic is that there is a sharp rising trend, largely common across categories, in the mean number of citations, per patent. This trend reflects, to a large extent, an increasing propensity to cite by PTO officers, as a result of the easier access to larger data sources brought about by computerisation of the PTO during the 1980's. Although the rising citations trend may not be *entirely* a pure artifact of the changed PTO practices, in the absence of a better alternative, the construction of the connection matrix for the second window was carried out using discounted citations data. In particular, the number of citations made by patents issued in class xy in the second window, was discounted by the xy growth rate of citations-made per patent between the first and second window.

There is a third potentially distorting characteristic in the data set, namely, the rising trend in the yearly number of patents issued since 1983. This feature is at least partly taken care of by our procedure, since according to our estimate of the connection matrix, the number of citations made by class xy patents, issued in window $[t + s, t + z]$, to class hk patents issued in $[t, t + z]$, is divided by the number of hk patents granted in $[t, t + z]$.

4.1 Modularity of empirical connection matrices

Fig. **??** and **??**, report a visual representation of the connection matrices $C(W1)$ and $C(W2)$ for the two windows. The colours identify different orders of magnitude of the connection coefficients.

The bright colour blocks and stripes depicted in Fig. 1 and 2 is partly revealing. For instance, the red main diagonal results from the fact every class tends to be more tightly connected with itself than with other classes; the same should apply to 'well chosen groups' of technology classes. The problem revealed by Fig. 1 and 2 is that the ordering of rows and columns is not particularly well chosen; it simply reflects the NBER original ordering of 3-digit classes, which is strongly influenced by temporal sequence in which the classes were first introduced. As a result, these figures do not offer an adequate visual representation of the quasi-modular structureof the two matrices. A far better candidate in this respect appears to be an endogenous permutation of the ordering that groups together of the classes showing a similar structural relationship with the other classes. To this end, we generated for each period a 32 groups partition of the 418 classes, and a corresponding permutation of C, using the algorithm CONCOR. The effects on the visual representation of connection strengths and their quasi-modular organization is quite sharp.

Table **??** and **??** specify the class composition of the CONCOR groups for the windows $W1$ and $W2$. The colours emphasise the correspondence between the endogenously generated groups and the NBER technological categories. For ease of later reference, and for reasons that will be clarified in the sequel, group 28 in Table **??** and group 1 in Table **??** are referred to as the 'Core-Groups' for the periods $W1$ and $W2$, respectively.

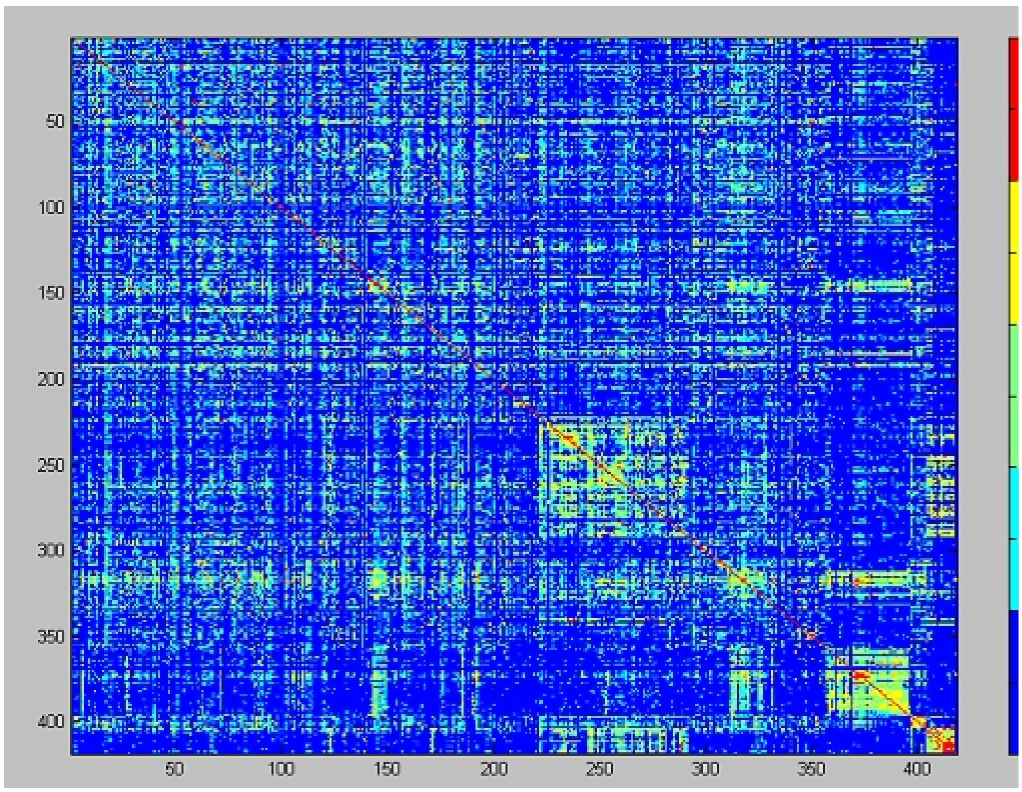


Figure 1: Average citation flows of 1 patent issued in a column technology-class, by patents issued in the row technology-class: 1975-1986. Representation based the NBER ordering of 418 3-digit classes. The colour sequence blue, light-blue, light green, yellow, red identifies progressively higher orders of magnitude of link intensity.

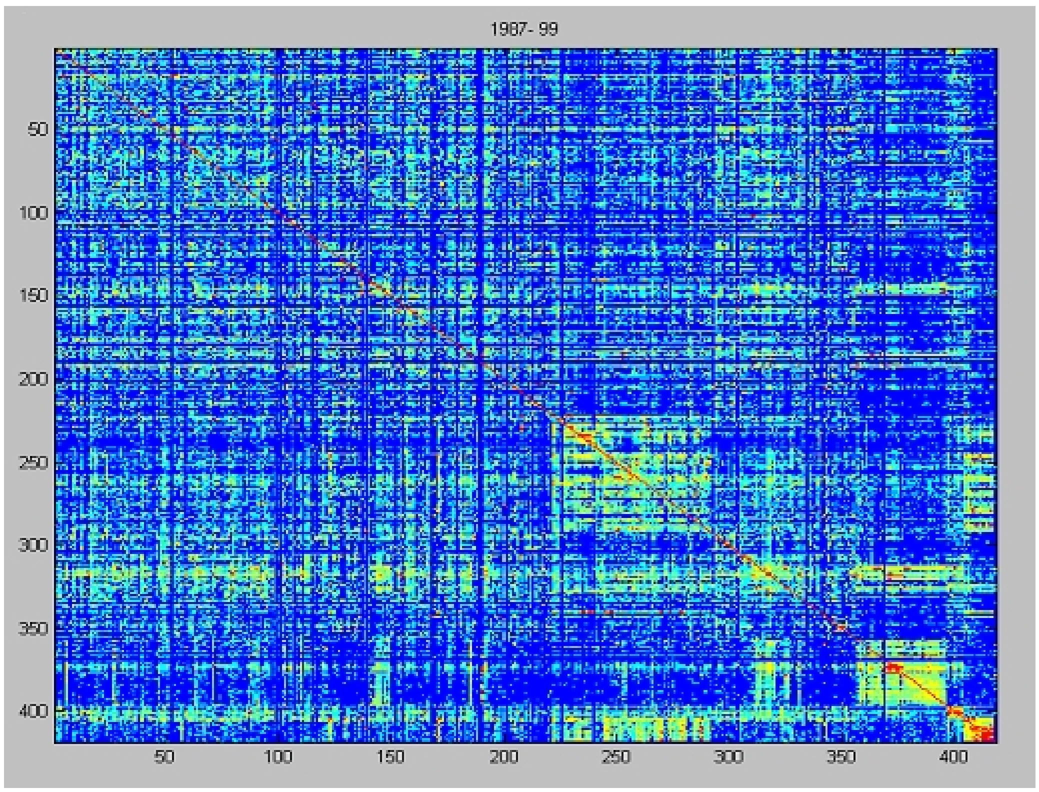


Figure 2: Average citation flows of 1 patent issued in a column technology-class, by patents issued in the row technology-class: 1987-1999. Representation based on the NBER ordering of 418 3-digit classes. Correspondence between colour and link intensity as specified in 1.

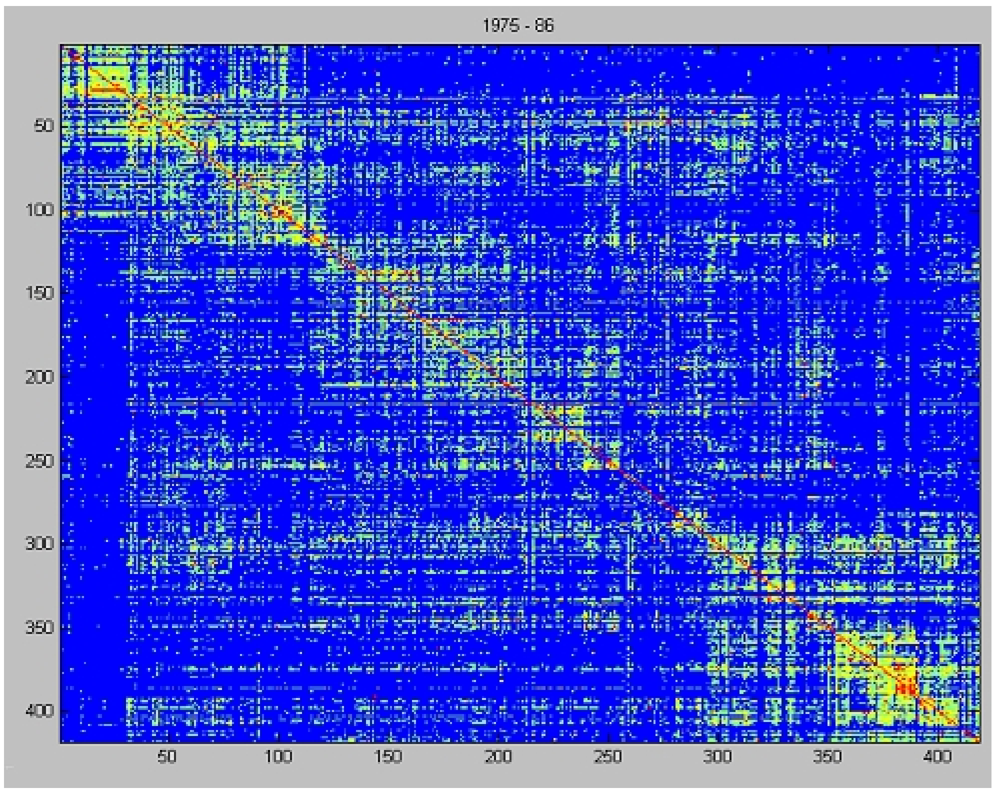


Figure 3: Average citation flows of 1 patent issued in a column technology class, by patents issued in the row technology-class: 1975-1986. Representation based the permutation of $C(W1)$ generated by the algorithm CONCOR. Colours identify link intensity as in Fig. 1 .

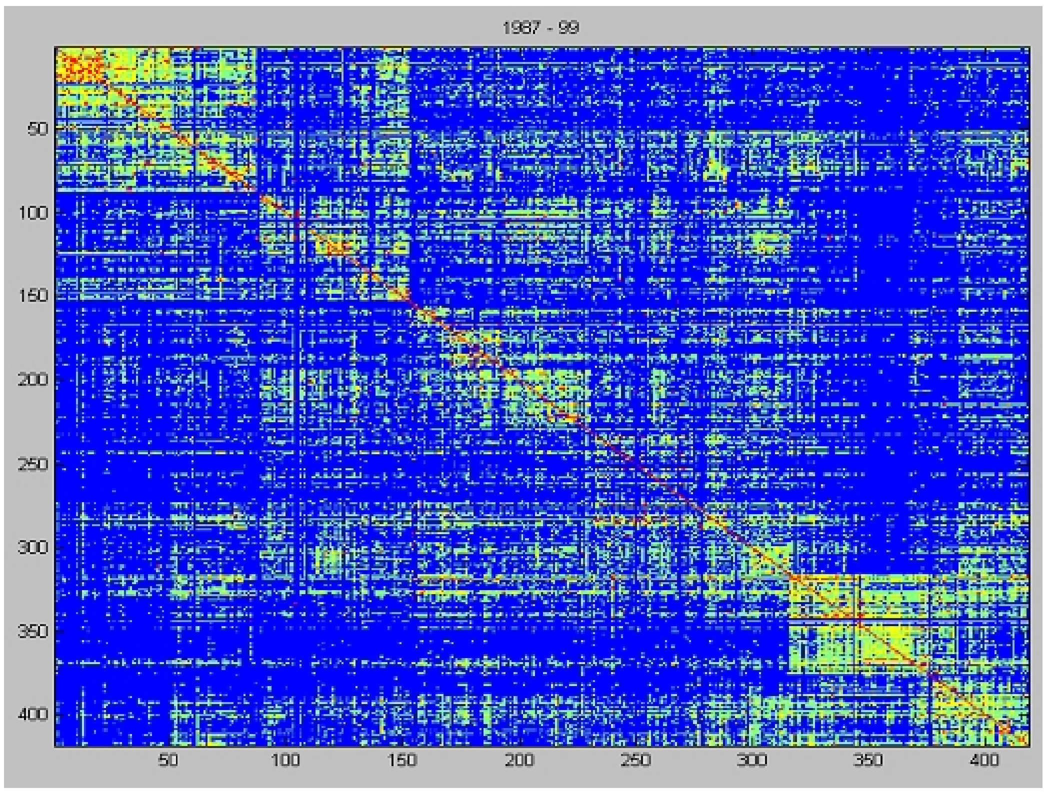


Figure 4: Average citation flows of 1 patent issued in a column technology class, by patents issued in the row technology-class: 1975-1986. Representation based the permutation of $C(W1)$ generated by the algorithm CONCOR. Colours identify link intensity as in Fig. 1

1: 1 132 260 507 508 510 512 516 526 556 568 570

2: 424 504 514 520 530 534 536 540 544 546 548 549 552 554 558 560 562 564

3: 8 28 51 57 106 152 156 162 252 264 351 427 428 430 433 442 501 503 521 522 523 524 525 527 528 604

4: 26 65 68 75 117 118 136 148 149 164 204 205 216 419 420 425 429

5: 15 23 55 71 95 96 127 131 134 159 166 209 210 241 261 366 422 426 435 436 494 588 800

6: 48 196 201 203 208 423 502 518 585

7: 34 44 99 110 202 266 373 432 449

8: 62 122 126 165 237 239 431

9: 2 4 5 36 54 119 124 128 141 472 473 482 601 602

10: 16 24 40 43 47 63 70 108 109 150 190 206 211 215 217 220 222 223 224 229 232 248 276 281 292 312 383 402

11: 14 27 49 52 114 135 160 169 181 182 238 249 256 293 296 403 404 405 411 441 454

12: 37 59 89 91 104 105 111 137 172 186 188 193 212 213 254 267 280 294 295 297 298 305 384 406 410 414 440

13: 7 29 30 72 76 79 81 82 83 125 140 142 144 147 173 227 228 269 279 301 407 408 409 413 451 470 483

Figure 5: Partition of the set of 3-digits classes into 32 structurally 'similar' groups performed by the algorithm CONCOR on the connection matrix $C(W1)$: groups 1-13. Blue = Computer and communic., Green = Mechanical, Pink = Chemical, Red = Drugs and medical, Yellow = Electrical and electronics, White = Others.

14: 60 86 92 102 175 184 192 251 277 285 291 299 415 416 417 418 464

15: 12 19 38 66 69 87 138 139 157 163 174 245 283 300 392 401 412
450 462 474 492 606 623

16: 53 56 100 112 171 198 221 225 226 242 270 271 289 452 460 493

17: 33 73 116 168 324 338 343 367 374 376 600 607

18: 219 257 313 362 372 385 438 445

19: 200 218 231 335 336 337 439 505

20: 278 307 310 314 315 318 320 322 323 361 363 388

21: 42 244 446

22: 187 191 258

23: 74 180 185 475 476 477

24: 123 236 246 290 303 701

25: 84 326 327 330 341 368 377

26: 329 331 332 333 334 342 375 380 381 455

27: 177 178 235 340 345 379 702 704 708

28: 365 370 395 700 705 706 707 709 710 711 712 713 714

29: 101 346 347 355 358 399 400

30: 250 348 349 352 353 356 359 378 382 386 396

31: 194 199 234 360 369 453

32: 273 434 463

Figure 6: Partition of the set of 3-digits classes into 32 structurally 'similar' groups performed by the algorithm CONCOR on the connection matrix $C(W1)$: groups 14-32. Blue = Computer and communic., Green = Mechanical, Pink = Chemical, Red = Drugs and medical, Yellow = Electrical and electronics, White = Others.

1: 1 340 345 365 370 379 380 395 700 702 704 705 706 707 708 709 710 711 712 713 714
2: 84 178 186 194 235 273 341 348 358 360 368 382 386 434 463
3: 323 326 327 330 331 333 334 377
4: 329 332 342 375 381 455
5: 33 73 147 250 324 351 356 367 374 376 378 600 607
6: 136 257 313 315 359 361 362 372 385 438 445
7: 270 271 346 347 355 399 400
8: 349 352 353 369 396 453
9: 37 56 104 105 172 212 213 278 298 414 460
10: 42 114 124 188 199 242 244 248 258 276 294 343 446 482
11: 60 91 102 280 440
12: 74 123 180 192 291 303 475 476 477 701
13: 70 116 185 191 292
14: 200 218 231 335 336 337 338 439
15: 177 187 236 246
16: 290 307 310 318 320 322 363 388
17: 2 12 26 28 36 38 54 57 66 87 128 156 168 245 425 450 602

Figure 7: Partition of the set of 3-digits classes into 32 structurally 'similar' groups performed by the algorithm CONCOR on the connection matrix $C(W2)$: groups 1-17. Colours identify NBER 1-digit Categories, as in 5 and 6.

18: 40 47 53 141 150 190 206 215 217 220 221 222 223 229 281 283 383 401 402 412 452 462 493

19: 4 5 16 24 43 89 108 119 211 224 232 267 296 297 312 410 441 473 601

20: 14 27 49 52 109 135 160 169 182 238 249 256 293 403 404 405 449 454 472

21: 7 30 51 59 63 69 76 79 81 82 86 125 140 142 144 157 163 164 173 174 193 225 227 228 234 254 269 279 289 295 300 301 305 407 408 409 411 413 429 470 483 492

22: 19 29 72 83 100 101 112 198 216 219 226 314 373 392 419 451 606

23: 15 111 139 152 171 175 299 420 474

24: 92 137 138 165 181 184 239 251 277 285 384 406 415 416 417 418 464

25: 8 106 162 264 427 428 430 433 442 501 503 505 522 604 623

26: 149 166 252 260 502 507 508 520 521 523 524 525 526 527 528 556 585

27: 44 424 504 514 530 534 536 540 544 546 548 549 552 554 558 560 562 564 568 570

28: 71 127 132 426 435 436 510 512 516 800

29: 23 48 159 196 201 202 203 208 423 518 588

30: 55 62 68 95 96 134 209 210 241 261 366 422 494

31: 34 99 110 122 126 131 237 431 432

32: 65 75 117 118 148 204 205 266

Figure 8: Partition of the set of 3-digits classes into 32 structurally 'similar' groups performed by the algorithm CONCOR on the connection matrix $C(W2)$: groups 18-32. Colours identify NBER 1-digit Categories, as in 5.

Newman and Girivan [21] propose that the appropriateness of any two community-structure partitions of a given network are evaluated using their proposed measure of modularity Q. This suggests that the the 32-blocks endogenous partition generated by CONCOR identifies a community structure moderately better than the NBER technological partition in 36 subcategories[8]. In period $W1$ the Newman-Girivan Q measure of modularity ([20], [18]) moderately increases for the former ($Q = 670135$), with respect to the latter ($Q = 647874$). The corresponding increase of the Q measure is lower for period $W2$.($Q = 614495$ against $Q = 610734$). The first point to observe is that, irrespective of the community structure adopted, the aggregate Q measure of modularity declines from the period 1975-1986 to the period 1987-1999.

The group-contributions to modularity, weighted and unweighted by the number of group members, is reported in Table 9. What is most relevant in this table (recall that the group composition changes from $W1$ to $W2$) is that in both periods the maximum per-class contribution to modularity comes from the blue coloured 'Core Group' comprising (almost[9]) exclusively classes in the Computer and communications technological category.

Correspondingly, in the list of the contributions to modularity induced by the exogenous partition of classes into 36 subcategories (Table 10) the blue coloured Computer and communications subcategories rank very high[10].

The analysis above, suggests that the technology classes in the Computer and communication technological subcategories, and even more the ICT (Information and communication technology) classes belonging to the 'Core Group' are not only most active in R&D, but receive and send a much higher than average share of their citations from and to classes belonging to the same subcategory or group. Apparently this conclusion marks a sharp contrast with the finding in [11], based on the Herfindahl concentration of the *class* distributions of patent citations made (input) and received (output). Hall, Jaffe and Trajtenberg [11] find that, on average, and throughout the period 1975-1999, patents in the Computer and communications category, have the *lowest concentration indexes of the input and output patent citations by class*. On this account, they argue that patents in Computer and communications are most 'original' because they creatively exploit knowledge from a wider set of technology classes, and produce also the most 'general' knowledge, because knowledge created by them disseminates to a wider set of classes According to [11],the highest generality score makes the label 'general purpose technologies' most appropriate for the classes belonging to the Computer and communications category.

The solution to the apparent paradox is that the modularity measures considered in this paper are based on the grouping together of classes into 'similar' technological communities. The fact that the ICT classes exhibit a relatively high modularity measure, based on this partition, does not contradict the fur-

[8] It may be worth observing that this partition gives rise to a class ordering which is not the NBER 'historical' ordering embedded in Fig. 1.

[9] The qualification in parenthesis refers to the period 1987-1999.

[10] The correspondence between colour and technological category is reported in previous tables.

Modularity: class partition by structural similarity (CONCOR)

	1975-1986			1987-1999		
Block	Q(i)	100Q(i)/n(i)	n(i)=size i	Q(i)	100Q(i)/n(i)	n(i)=size i
1	0,015216	0,0849731	12	0,067554	0,1976741	21
2	0,026971	0,1004123	18	0,028377	0,1162501	15
3	0,055041	0,141865	26	0,012253	0,0941175	8
4	0,026505	0,1044819	17	0,010877	0,1113976	6
5	0,039946	0,1163879	23	0,021183	0,1001295	13
6	0,019552	0,1455831	9	0,022052	0,1231894	11
7	0,011816	0,0879813	9	0,014192	0,1245844	7
8	0,012821	0,12274	7	0,0094745	0,0970338	6
9	0,022328	0,1068769	14	0,012394	0,0692368	11
10	0,03595	0,0860405	28	0,019002	0,0834045	14
11	0,030036	0,0958484	21	0,0066155	0,0813038	5
12	0,033801	0,0838934	27	0,020869	0,1282389	10
13	0,035748	0,0887259	27	0,0038999	0,0479293	5
14	0,024742	0,0975322	17	0,0092108	0,0707498	8
15	0,025293	0,0736944	23	0,004089	0,0628167	4
16	0,023255	0,0973999	16	0,01374	0,1055394	8
17	0,021061	0,1176143	12	0,020218	0,0730815	17
18	0,016614	0,1391703	8	0,033912	0,0906032	23
19	0,008697	0,072852	8	0,027984	0,0905054	19
20	0,018281	0,1020895	12	0,024231	0,0783675	19
21	0,0034015	0,0759821	3	0,038334	0,0560858	42
22	0,0023823	0,0532154	3	0,01974	0,0713537	17
23	0,0094076	0,1050727	6	0,010216	0,069752	9
24	0,011247	0,1256168	6	0,022459	0,081182	17
25	0,012458	0,1192649	7	0,026703	0,1093923	15
26	0,017534	0,1175015	10	0,029153	0,1053786	17
27	0,016086	0,1197754	9	0,020944	0,0643499	20
28	0,041107	0,2119018	13	0,012898	0,0792575	10
29	0,017668	0,169142	7	0,010113	0,0564944	11
30	0,02227	0,1356719	11	0,020976	0,0991511	13
31	0,007173	0,0801146	6	0,011306	0,0771942	9
32	0,0057265	0,1279176	3	0,0095249	0,0731625	8
Total	0,670135		418	0,614495		418

Figure 9: Group contributions to modularity, unweighted and weighted by group size, based on the two different community structures identified by CONCOR for the periods 1975-1986 and 1987-1999.

Modularity: class partition by subcategory

	1975-1986	1987-1999
subcat.	Q(i)	Q(i)
11	0,012064	0,0083618
12	0,011066	0,0098523
13	0,020171	0,016662
14	0,011017	0,0068588
15	0,023214	0,018136
19	0,020158	0,016958
21	0,020131	0,022677
22	0,027234	0,031
23	0,026664	0,023277
24	0,020847	0,023604
31	0,012689	0,012595
32	0,030077	0,03077
33	0,016459	0,015226
39	0,020641	0,016632
41	0,014486	0,01609
42	0,015028	0,017048
43	0,015384	0,014143
44	0,01641	0,014861
45	0,018402	0,017237
46	0,028925	0,030943
49	0,015925	0,015967
51	0,015154	0,013566
52	0,013972	0,011623
53	0,020369	0,017744
54	0,02023	0,018484
55	0,01539	0,015657
59	0,012985	0,013803
61	0,01739	0,015989
62	0,016927	0,017526
63	0,015216	0,014982
64	0,023013	0,018805
65	0,014843	0,015793
66	0,019905	0,013316
67	0,014378	0,012687
68	0,016992	0,018203
69	0,014118	0,013657
Total	0,647874	0,6107339

Figure 10: Group contributions to modularity according to the NBER partition of 418 3-digit classses into 36 2-digits subcategories.

ther fact that they also rank lowest in the Herfindhal concentration index based on the class distribution (of inward and outward citations). It simply means that a relatively large share in the 'wider sets of technology classes' sending knowledge connections to and receiving knowledge connections from the ICT classes, belong in the same technology group or subcategory. The qualification by Hall, Jaffe and Trajtenberg [11] of the ICT classes as 'general purpose technologies' is therefore inappropriate, if it is simply based on their finding concerning the Herfindahl index. To corroborate this conclusion, we must spel the doubt that the apparent clash between our modularity measures and the cited results of [11] may have to with the fact that the former, unlike the latter, are based on the connection matrices $C(W1)$ and $C(W2)$ in which each column distribution of the citations receaved by one class is normalized by the number of patents issued in that class in the corresponding period. To this end, we report below the Herfindahl concentration indexes concerning the distribution of the absolute number of inward and outward citations, by group and subcategory, for the periods $W1$ (Table **??**) and $W2$ (Table **??**). The findings corroborate our modularity result, confirming that the Computer and communications subcategory, and most prominently, the 'Core Group' *rank relatively high in the ordering of concentration indexes*.

Our findings do not necessarily contradict the idea that the new knowledge embodied in ICT innovations was 'general purpose' and, as such, could be exploited in a wide set of diverse technology classes. The corroboration of this idea can not simply rest on concentration indexes of citation distributions by class. As will be shown, it requires a much more elaborate analysis of some structural properties of the connection matrices, focused on the notions of near-decomposability, and autocatalytic sets.

It is to this analysis that we now turn.

4.2 Near decomposability and the Core properties of ICT

The partitions of the connection matrices $C(W1)$ and $C(W2)$ into blocks performed by CONCOR (see Fig.s 5 - 8) share the property that, in both $W1$ and $W2$, the technological community, or group, exhibiting the highest Q modularity measure is composed (almost) exclusively by ICT classes. It is now time to justify the claim that each of the two communities thus identified represents the 'Core group' (CG) in the partition for the period $W1$ and $W2$, respectively, and to discuss the relevance of this claim.

The first step in the argument is to see to what extent each CG represents a module in the sense of Simon [32], that is, in the sense that the size of the connection links between the classes within the group are at least one order of magnitude larger than those sent to, or receaved from, the classes that do not belong in the group. To this end, we produced two dichotomized connection matrices $C_A(W1)$, $C_A(W2)$ with the defining property that all connection links of the original matrix $C(Wi), i = 1, 2$, that are larger than or equal to 0.1 are set equal to 1 and all the others are set to 0. The exercise shows that in both $W1$ and $W2$, the CG is connected by first-order-magnitude links with classes

H* on unweighted 32x32 matrix

block	input distr.	output distr.
1	0,38717	0,35993
2	0,7087	0,64151
3	0,5593	0,59829
4	0,48597	0,46876
5	0,50805	0,53951
6	0,56613	0,55953
7	0,3368	0,37148
8	0,50963	0,52178
9	0,52957	0,58593
10	0,55445	0,51401
11	0,51822	0,51927
12	0,50702	0,46333
13	0,47776	0,50606
14	0,48874	0,45484
15	0,38458	0,35475
16	0,5528	0,52312
17	0,48688	0,50091
18	0,47909	0,51145
19	0,60956	0,53109
20	0,47802	0,44215
21	0,41756	0,44173
22	0,37798	0,46299
23	0,4235	0,3848
24	0,50749	0,62645
25	0,5	0,43053
26	0,49958	0,46926
27	0,33754	0,34883
28	0,47518	0,52802
29	0,49896	0,61149
30	0,54692	0,5122
31	0,52156	0,53522
32	0,50171	0,57316

H* on 36x36 unweighted matrix

subcat.	input distr.	output distr.
11	0,2405	0,28089
12	0,19727	0,20308
13	0,38906	0,38557
14	0,57121	0,35733
15	0,52319	0,48689
19	0,49983	0,51596
21	0,49434	0,49226
22	0,33026	0,45034
23	0,31244	0,36196
24	0,50297	0,45282
31	0,35729	0,48437
32	0,53607	0,63031
33	0,38477	0,42907
39	0,51528	0,55276
41	0,50032	0,3742
42	0,51939	0,47803
43	0,41635	0,39916
44	0,42619	0,37802
45	0,46774	0,44189
46	0,52496	0,52765
49	0,33447	0,35676
51	0,46315	0,44653
52	0,39081	0,44235
53	0,56713	0,56178
54	0,50677	0,55653
55	0,50329	0,5281
59	0,41762	0,37756
61	0,57993	0,61724
62	0,57877	0,66926
63	0,59548	0,603
64	0,56644	0,64103
65	0,46747	0,50109
66	0,48288	0,52523
67	0,3641	0,28828
68	0,50343	0,49064
69	0,36531	0,37321

Figure 11: Normalized Herfindahl concentration indexes concerning the distributions of patent citations made (input distribution) and received (output distribution) by each group in the 32 and 36 group partitions for the period 1975-1986.

H* on unweighted 32x32 matrix

block	input distr.	output distr.
1	0,55179	0,62112
2	0,49952	0,45803
3	0,49585	0,38539
4	0,40006	0,42929
5	0,47073	0,44328
6	0,50928	0,52245
7	0,52574	0,54867
8	0,44905	0,4655
9	0,4388	0,44783
10	0,41861	0,43937
11	0,44019	0,43809
12	0,54338	0,55387
13	0,49401	0,48727
14	0,54077	0,44595
15	0,30394	0,28059
16	0,37925	0,40569
17	0,31749	0,34243
18	0,50404	0,48005
19	0,50726	0,5056
20	0,44991	0,47037
21	0,40343	0,35045
22	0,32057	0,34701
23	0,42794	0,36886
24	0,4514	0,3756
25	0,36602	0,39246
26	0,47451	0,43047
27	0,5949	0,60781
28	0,38655	0,42523
29	0,34918	0,30634
30	0,39227	0,43734
31	0,4445	0,42645
32	0,31926	0,2845

H* on 36x36 unweighted matrix

subcategory	input distr.	output distr.
11	0,17672	0,17323
12	0,17233	0,16366
13	0,37932	0,32954
14	0,33494	0,25759
15	0,41147	0,40042
19	0,42455	0,41261
21	0,50328	0,45919
22	0,38226	0,55381
23	0,29912	0,38995
24	0,50524	0,4704
31	0,40314	0,47732
32	0,6095	0,61185
33	0,32431	0,47084
39	0,43435	0,39762
41	0,43571	0,36686
42	0,50844	0,45073
43	0,35293	0,32019
44	0,33316	0,31895
45	0,39287	0,38283
46	0,4924	0,6228
49	0,33032	0,28123
51	0,37365	0,33923
52	0,28814	0,30873
53	0,51518	0,52843
54	0,43445	0,47727
55	0,49357	0,46358
59	0,38952	0,32853
61	0,50637	0,52189
62	0,60722	0,58737
63	0,53646	0,50044
64	0,49248	0,58134
65	0,46961	0,46472
66	0,4208	0,38367
67	0,34455	0,25277
68	0,46893	0,40769
69	0,31904	0,32433

Figure 12: Normalized Herfindal concentration indexes concerning the distributions of patent citations made (input) and received (output) by each group in the 32 and 36 partitions for the period 1987-1999.

that do not belong in the group; hence it does not meet the strong requirements imposed by Simon's definition of a module.

4.2.1 The dominant autocatalytic set of C_A

Simon's idea of separating first-oder-magnitude from lower-order magnitude links brings to the fore interesting functional and structural properties of the connection matrix C. It turns out that the dominant ACS of the dichotomized connection matrix $C_A(W1)$ has a Core consisting of 8 classes, all of which are in the Computer and communication category, and all belonging to the CONCOR community CG($W1$), which drwas its name from this finding (see 13). They are:

705 Data processing: financial, business practice, management, or cost/price determination

707 Data processing: database and file management or data structures

709 Electrical computers and digital processing systems: multicomputer data transferring

710 Electrical computers and digital data processing systems: input/output

711 Electrical computers and digital processing systems: memory

712 Electrical computers and digital processing systems: processing architectures and instruction processing (e.g., processors)

713 Electrical computers and digital processing systems: support

714 Error detection/correction and fault detection/recovery

Three out of the five classes in CG($W1$), which do not belong to the core of the dominant ACS($C_A(W1)$), belong to its periphery. They are:

365 Static information storage and retrieval

370 Multiplex communications

700 Data processing: generic control systems or specific applications.

Finally, the remaining two classes of CG($W1$), namely, class 706 (artificial intelligence) and 395, do not belong to the dominant ACS($C_A(W1)$), but send first-order magnitude links to members of this set.

The set of nodes in the periphery of the dominant ACS($C_A(W1)$) consists of 37 classes, 17 in the Computer & communications category, 15 in Electrical & electronics, 3 in Others and 2 in Mechanical. Figure **??** offers a visual representation of the link architecture of the dominant ACS($C_A(W1)$). The 8 blue nodes of the core send first order magnitude links not only to nodes allined on a one-way path, but also to 8 other loops of strongly connected components that are core structures in smaller ACS embedded in the periphery of the dominant ACS($C_A(W1)$) (4 components with 2 members, 2 with 4 members, 2 with 5 members).

The number of classes in the dominant ACS($C_A(W2)$) is not much larger than the corresponding number in the dominant ACS($C_A(W1)$): 52 in the former against 45 in the latter. It is the relative composition of the dominant ACS between core and periphery to change from period $W1$ to $W2$. The Core of the dominant ACS($C_A(W2)$) contains 42 member classes, of which 28 belong to

Autocatalytic set of C(W1.A)

Core : 8 members

705 707 709 710 711 712 713 714

Periphery: 37 members

73 101 177 181 235 250 257 313 315 318 324 327 340 341 345
346 347 348 356 358 360 365 369 370 375 377 379 380 381 382
386 400 438 455 700 702 704

Autocatalytic set of C(W2.A)

Core : 42 members

180 192 235 303 318 324 326 327 340 341 345 347 348 358 360
365 369 370 375 379 380 382 386 395 399 400 455 475 477 700
701 702 705 706 707 708 709 710 711 712 713 714

Periphery: 10 members

29 73 74 123 257 280 310 361 438 439

Figure 13: Core and Periphery of the Autocatalytic sets of the connection matrices $C(W1)$ and $C(W2)$. Blue = Computer and Communications, Yellow = Electrical and electronics, Green = Mechanical.

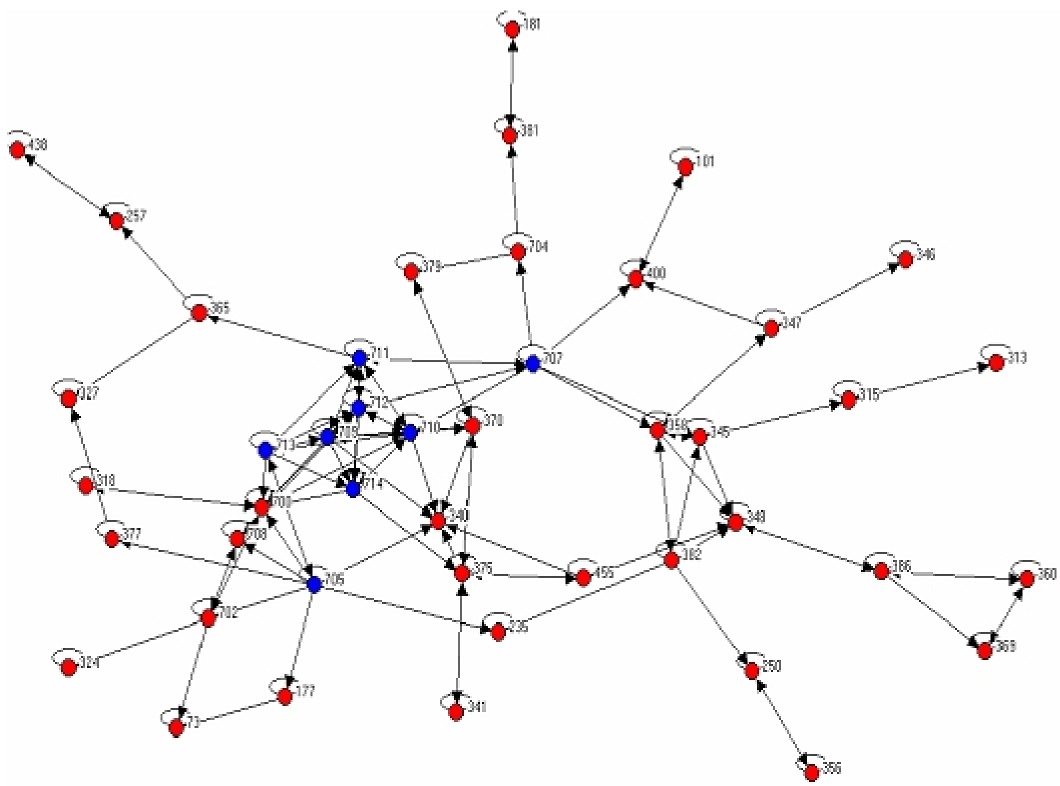


Figure 14: Dominant ACS of the dichotomized matrix $C_A(W1)$. The blue and red nodes correspond to core and periphery nodes, respectively.

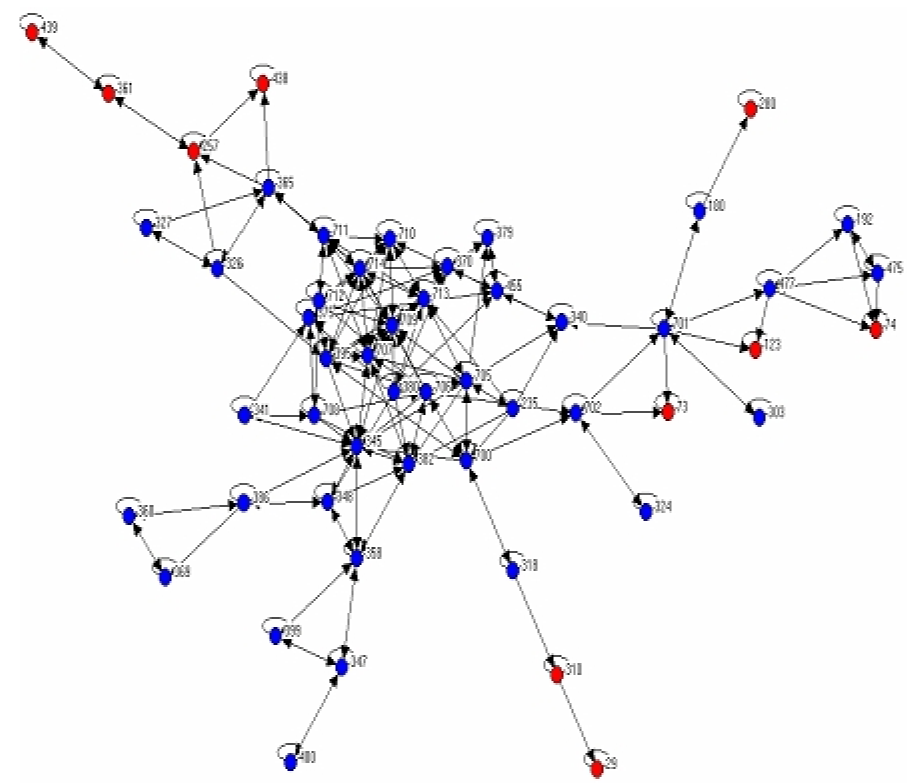


Figure 15: Dominant ACS of the dichotomized matrix $C_A(W2)$. The blue and red nodes correspond to core and periphery nodes, respectively.

Computer & communications, 8 to Mechanical and 6 to Electrical & electronics. The sharp absolute and relative increase in the number of core members in the second period is illustrated in 13. The increase in core size and its more differentiated composition by technological category signals a higher degree of integration of the ITC dominant paradigm with the rest of the economy. A much larger number of classes belonging to more heterogeneous technologies, is participating beside the core ITC classes in the first-order size self-sustaining mechanisms of knowledge creation and transmission in period $W2$ as compared to $W1$. Figure **??** shows the changed structure of the dominant ACS ($C_A(W2)$). Most loops of strong components previously embedded in the periphery of the dominant ACS have now been included in the new expanded core. There are only 10 nodes in the periphery, all belonging to the categories Mechanical and Electrical & electronics; of them, 4 form a strongly connected component.

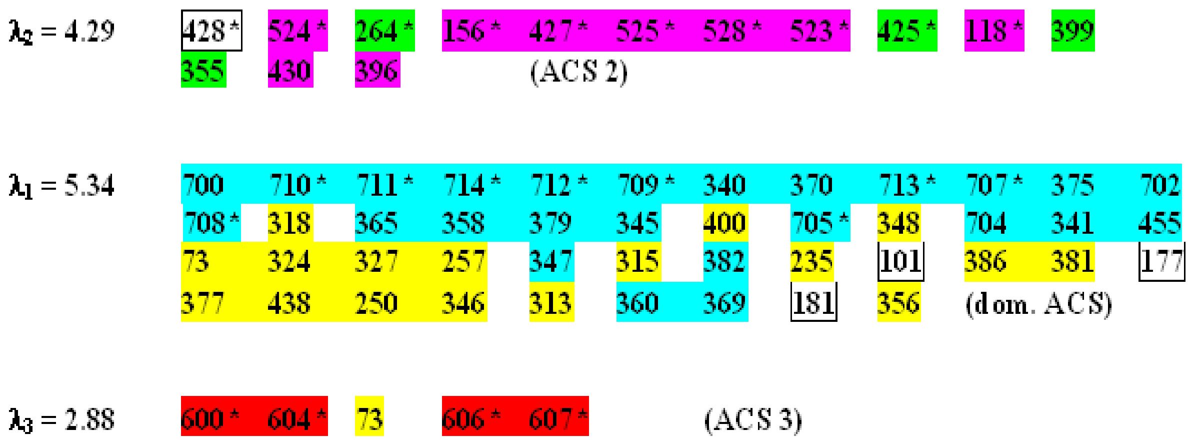


Figure 16: Class composition of the 3 autocatalytic sets of the matrix $C_A(W1)$ associated with the first 3 highest positive eigenvalues. Autocatalytic sets embedded in dom ACS have not been considered. Classes with the asterisc are in the core of the ACS.

4.3 Non dominant Autocatalytic sets of C_A

The directed networks corresponding to the dichotomized matrices $C_A(W1)$ and $C_A(W2)$ embed two other prominent, but much smaller autocatalytic sets, beside the dominant one. Their class composition in the first period is shown in Figure 16, where they are labelled ACS_2 and ACS_3, respectively. They are associated to positive eigenvalues of the matrix $C_A(W1)$, namely $\lambda_2 = 4.29$ and $\lambda_3 = 2.88$. The functional significance of the ACS concept in the context of patent analysis is confirmed by the fact that, as was the case for the dominant ACS, both $ACS_2(W1)$ and $ACS_3(W1)$ correspond to functional groups of classes that lend themselves to a clear interpretation.

In particular, $ACS_2(W1)$ comprises 14 classes that bear a close relation with the category Chemicals. With the possible sole exception of class 418 (Stock material or miscellaneous articles), the 10 classes in the core of $ACS_2(W1)$ either belong in this category, or identify processes that use plastics as their material support.

$ACS_3(W1)$ is considerably smaller and refers to more specialized classes of activity related to surgery, in the category Drugs & medicals. It may be worth observing that $ACS_3(W1)$ and the dominant $ACS(C_A(W1))$ have one node of their respective peripheries in common, namely the node corresponding to class 73 (Measuring and testing) in the category Electrical & electronics. Thus, not even the concept of ACS identifies functional modules that fully correspond to Simon's definition of a module.

The two autocatalytic sets ACS_2 and ACS_3 are also clearly detectable in the second period, that is, in the network corresponding to the dichotomized matrix $C_A(W2)$. The expansion of first order magnitude links in the second period is not circumscribed to the dominant ACS, but is a more general trend. ACS_2 expands from 14 to 20 members and marks a pronounced change in its composition. 4 classes exit and 10 new classes enter the ACS_2, mostly belonging in the categories Chemicals and Electrical & electronics. The outcome is also a change in the composition of the core, which in the second period consists of 9 classes, all of which in Chemicals, and only 4 of them already part of the core in the first period.

The expansion of ACS_3 from the first period to the second is much sharper than for the other autocatalytic sets. The number of members increases from 5 to 24 nodes, with new classes entering the core and the periphery, while no exits occur. The expanded core now contains 6 classes, again highly specialized and all related to the activity of surgery or fabrication of prosthesis. In spite of the highly specialized core, all technological categories except Computer & communications are represented in this autocatalytic set in period $W2$. The member nodes include classes in the semiconductor technology, in molecular biology, in chemistery and in mechanical processes using plastics as material support. It is also remarkable that the $ACS_2(C_A(W2))$ shares with the $ACS_3(C_A(W2))$ 10 members of its perifery (belonging in the categories Chemicals, Electr. & electronics, Mechanicals and 'Others') and that both the $ACS_2(C_A(W2))$ and the $ACS_3(C_A(W2))$ share 4 nodes with the dominant $ACS(C_A(W2))$, all of which in Electrical & electronics. This finding strongly reinforces the conclusion that autocatalytic sets are functional structures that do not correspond to modules in Simon's sense.

As it turns out, the approximation based on first-order-magnitude links, standardized to weight size 1, detects structural properties of the empirical connection matrix $C(W)$ that do not lose their significance after all links of every magnitude have been re-introduced and all links bear their proper weight. From the weighted directed graph $G(S, L, C)$, corresponding to the empirical connection matrix C, we extract the subgraph $G(S_i, L_i, C_i)$ where S_i is the set of nodes (classes) in the ACS_i of the dichotomized matrix C_A defined above (the dominant ACS is ACS_1), $L_i \subset L$ is the subset of links connecting the nodes in S_i, and C_i is the connection matrix specifying the intensity of the links in L_i, as reported in C. Our model suggests that the relative degree of participation of a subset S_i of nodes (classes) to the long-term self sustaining mechanisms of knowledge creation and transmission within C can be evaluated by comparing the Perron-Frobenius eigenvalue of the connection matrix C_i of the subgraph $G(S_i, L_i, C_i)$ with the dominant eigenvalue of C. The results of this analysis, carried out for the periods $W1$ and $W2$ are as follows.

	$\lambda^*(C)$	$\lambda^*(C_1)$	$\lambda^*(C_2)$	$\lambda^*(C_3)$
$W1$	3.5825	3.5603	2.5081	2.8827
$W2$	6.6492	6.6442	3.3182	3.4045

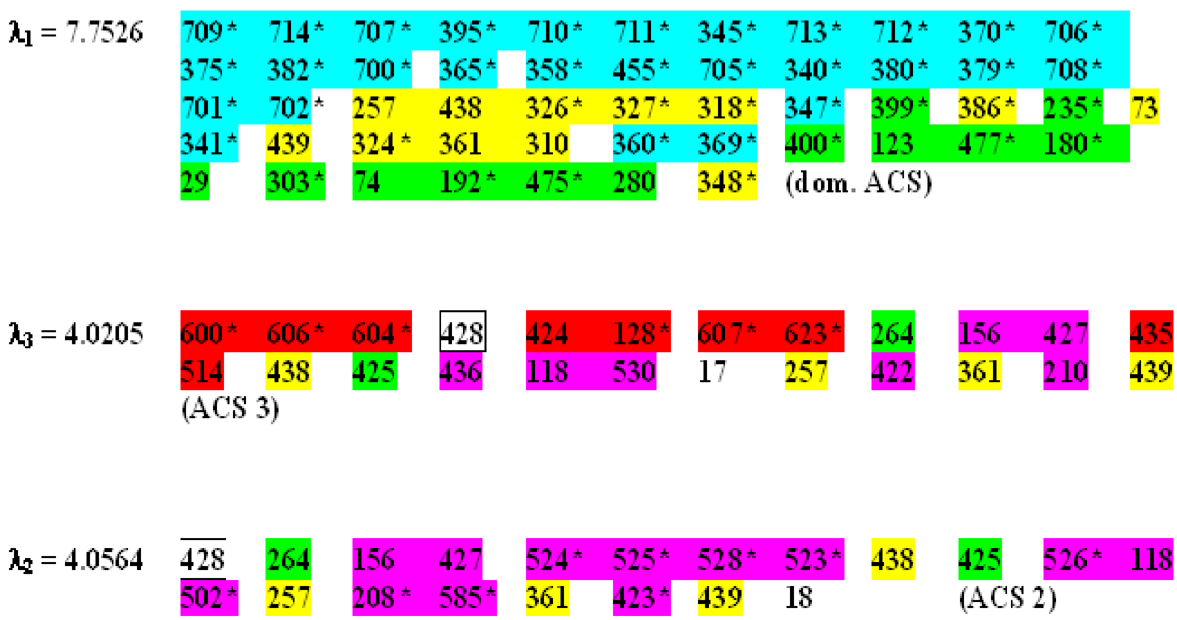


Figure 17: Class composition of the 3 autocatalytic sets of the matrix $C_A(W2)$ associated with the first 3 highest positive eigenvalues. Autocatalytic sets embedded in dom ACS have not been considered. Classes with the asterisc are in the core of the ACS.

5 Conclusions: ICT fields, general purpose technology, modularity, and ACS

Since our narrative is focused on the role played by the ICT fields during the period of analysis, it is worth summarising our findings in this perspective, and bring them within a coherent framework.

A first order of considerations refers to structural features of the knowledge transfer between technology fields that are common to the periods under consideration, 1975-1986 and 1987-1999.

If the organization of the learning interfaces connecting technology fields is regarded through the lenses of the Newman Girivan [21] Q measure of modularity, and if connection links of every size are considered, the organization concerning the ICT fields reveals unquestionably a relatively high degree of modularity. The statement must be interpreted in the sense that this technology group shows a maximum, or at least relatively high, propensity to be more tightly connected with itself than with other groups. As already observed, this result is confirmed by the finding that the distributions by technology 'group' (CONCOR generated group, or NBER subcategory) of inward and outward patent citations shows a relatively high Herfindahl concentration index for the ITC group compared to most of the others. In the light of these findings the frequent statement that the ICT represent the prominent general purpose technologies of late 20th century needs qualification and re-interpretation.

Our suggested interpretation is based on a separation of knowledge links according to their order of magnitude. By focusing on sufficiently strong links, and abstarcting temporarily from the others, we show that in both periods 1975-1986 and 1987-1999 a subset of ICT fields is a crucial part of a core sub-network connected by a circular path of strong links that together build a self-sustaining mechanism of knowledge transfer. The core structure pertaining to the critical ICT fields is not unique in the network of first order magnitude links and in period 1975-1986 it is not even the largest core structure with respect to the number of classes in it. What is special about it is: (i) It obtains, among all such structures, the highest rate of knowledge transfer, as evaluated through a dominant eigenvalue measure. (ii) It reaches out, through first order links, the by far largest periphery of technology classes, which together with the source core build up the dominant ACS(C_A) of the strong-links network. We emphasize that the closed path connectivity of the core implies that the systematic and persistent flow of knowledge transfer from the core of the dominant ACS(C_A) to the periphery and to the network at large, through second order links, contains a form of knowledge aggregation in the sense that the core fields participate in that transfer in relatively fixed proportions. In our view, these functional characteristics, as opposed to measures based on Herfindahl concentration indexes, or Q modularity, justufy the label of 'general purpose' for the ICT in the period 75-99.

When we ask what technologies may be or may not be regarded as general purpose in a given historical period, we are posing questions concerning the absolute pervasiveness of such technologies. The judgement cannot abstract from the absolute overall intensity of the links connecting the technology in question to the others. This is precisely what occurs when we measure, through the Q modularity, how community oriented is the organisation of a connection structure,independently of its overall intensity, or how concentrated is a distribution of connections (by using Herfindahl indexes).

A second order of considerations refers to the detection and euristic interpretation of structural change, if any, between the first and the second period. Also in this respect our narrative is focused on the core ICT fields.

On average, between 1975-86 and 1987-99 we register a sharp and general increase in the intensity of knowledge transfer across fields, in spite of the fact that: (i) the number of citations in the second period was discounted in the attept at eliminating distortions induced by the changed USPTO citing practices; (ii) the connection matrices $C(W1)$ and $C(W2)$ report the number of citations per unit of patent issued in the period. To some extent, the implied higher information flows were more diffused and less community oriented in the second period, as is suggested by a mild decline of the Q modularity measures from $W1$ to $W2$. In this respect the ICT group, no matter if identified by the NBER classification criteria or through 'structural similarity' criteria, simply follows the general trend.

As before, a different picture is obtained by focusing on first-order magnitude links. The number of nodes (classes) in the dominant ACS of the dichotomized connection matrix C_A increases from 45 in period $W1$ to 52 in period $W2$, a fact which is in line with the general trend referred to above. What is striking is the change from $W1$ to $W2$ in the composition of the dominant ACS(C_A) between core and periphery. In 1975-99 the 8 core classes all belong to the NBER category Computer & communications, and more specifically to Electrical computers and digital processing systems, Data processing, and Error detection/correction. In 1987-99 the core contains 42 nodes, 28 of which in Computer and communications, 8 in Mechanicals and 6 in Electrical and electronics. The ratio between the size of core and perifery is reversed: $\frac{\#Core}{\#Periphery}$ is 0.216 in $W1$ and 4.2 in $W2$.We suggest that the increase in the *relative* size of the core, compared to periphery, and its more differentiated composition by technological category, signals a higher degree of integration of the ITC dominant paradigm with the rest of the economy. This finding is prima facie consistent with a evolutionary interpretation of design standard formation. During the early phase of design standard formation technological change is more turbulent and there is a marked trade off between the knowledge gains that can be obtained through tighter links with R&D in other fields, on the one hand, and the increased complexity of R&D, on the other. An excess of connectedness makes finding a 'fit' solution or design on a ltechnological andscape more difficult, because landscape are constantly deforming as a result of the fact that convergence to a stable technology standard is not yet complete and the solutions provisionally identified in different fields may not be compatible. The nature of the trade off is drastically altered in favour of connectedness, after a set of mutually compatible standards has emerged. Now the relation between connectedness and landscape deformation is much waker. Correspondingly, the incentives for a relatively large set of strong mutual interactions, as we observe in the core of the dominant ACS(C_A) is stronger.

6 Appendix A: a model of incremental innovations

Aggregate R&D effort $\chi = \sum_j Q_j$ is not explained by the model. It is assumed to grow at the exogenous exponential rate γ.The flow of useful innovations in sector i depends on two factors, the effective R&D effort in this field, Q_i/A_i, and the repertoire of available ideas that are the 'building blocks' of R&D in field i; this repertoire corresponds to the knowledge flows $\sum_j c_{ij}A_j$ received by i through the active interfaces described by C. The stock A_i, $j = 1, ...n$, evolves according to the differential equation:

$$\dot{A}_i = \sigma \frac{Q_i}{A_i} \sum_j c_{ij} A_j = \sigma Q_i p_i(A) \qquad (5)$$

where σ is a parameter, $p_i(A)$ is the function $\sum_j c_{ij}A_j/A_i$ with $0 \leq c_{ij}$. Some points concerning expression (5) are worth stressing. The innovation flow depends on the effective R&D effort Q_i/A_i , rather than the absolute effort Q_i, to allow for the fact that a larger stock A_i makes innovation in i more complex, hence more R&D intensive. For the sake of simplicity, the expression abstracts from technological obsolescence, which can be introduced at a minor cost. c_{ij} is the generic element of the $n \times n$ matrix C. Replacing every $c_{ij} > 0$ in C with 1, and leaving every zero element of C unchanged, we obtain the adjacency matrix $\tilde{C}$ of the directed graph $G(S, L)$.

The R&D effort of sector i, namely Q_i,changes according to the dynamic equation:

$$\dot{Q}_i = \left[\rho \left(p_i - \frac{1}{n} \sum_j p_j \right) + \gamma \right] Q_i \qquad (6)$$

Let $a_i = A_i / \sum_j A_j$, and $r_i = Q_i/A_i$. Then

$$p_i(A) = \frac{\sum_j c_{ij}A_j}{A_i} = \frac{\sum_j c_{ij}a_j}{a_i} = f_i(a) \qquad (7)$$

where $a = (a_1, ..., a_n)'$ and $f(a) : K \to R^n_+$. Notice that a is so defined that it belongs to the $n-1$ dimensional simplex K in R^n, that is, $0 \leq a_i \leq 1$, $\sum_j a_j = 1$. Indeed, since $A_i \geq 1$, a_i will approach zero if and only if $\sum_j A_j$

goes to infinity with A_i finite. From (5) and (6) we obtain:

$$\dot{a}_i = \sigma \left[r_i \sum_j c_{ij} a_j - a_i \sum_h r_h \sum_j c_{hj} a_j \right] \tag{8}$$

$$\dot{r}_i = r_i \left[\gamma + (\rho - \sigma r_i) f_i(a) - \frac{\rho}{n} \sum_h f_h(a) \right] \tag{9}$$

The following notation is now introduced: for every row n-dimensional vector of real variables $(x_1, x_2, ..., x_n)$, the corresponding label x is the column vector $(x_1, x_2, ..., x_n)'$ and X is the diagonal matrix with the elements $x_1, x_2, ..., x_n$ on its main diagonal; moreover, z is the n dimensional unit column-vector $(1, 1, ..., 1)'$. Now for the column vectors $a = (a_1, ..., a_n)'$, $r = (r_1, ..., r_n)'$ we generate the corresponding diagonal matrices A, R. From the equations above we obtain the system of non-linear differential equations:

$$\dot{a} = \sigma \left[RCa - ar'Ca \right] \tag{10}$$

$$\dot{r} = R \left[(\rho I - \sigma R) f(a) + z \left(\gamma - \frac{\rho}{n} z' f(a) \right) \right] \tag{11}$$

Our primary goal in this section is to study the system dynamics and to relate it to the topological structure of the matrix C.

Proposition 5 *Let a^* be the right eigenvector of C associated with the Perron-Frobenius eigenvalue λ^*. Using a genericity argument, we can safely assume that λ^* has multiplicity 1 (Hirsch and Smale [12, pp. 153-157]). (a^*, r^*) is a stationary state of equations (10)-(11), where r^* is defined as follows. (i) If $a^* > 0$, $r^* = (\gamma/\sigma\lambda^*)z$. (ii) If $a^* \geq 0$, let $n_1 < n$ be the number of strictly positive components of a^*, and $n_2 = n - n_1$. Since in this case C is reducible, there exists a permutation matrix P such that:*

$$PCP' = \begin{bmatrix} C_{11} & C_{12} \\ 0 & C_{22} \end{bmatrix}$$

Here C_{11} is a $[n_1 \times n_1]$ non negative matrix, C_{22} is a $[n_2 \times n_2]$ non negative matrix, and $\lambda^*(C) = \lambda^*(C_{11}) > \lambda^*(C_{22})$. On the simplifying assumption that the right eigenvector of C_{22} associated with $\lambda^*(C_{22})$ is strictly positive, we can write:

$$r_i^* = \frac{\rho f_i(a^*) + \gamma - (\rho/n)\left(n_1 \lambda^* + n_2 \lambda^*(C_{22})\right)}{f_i(a^*)\sigma} \tag{12}$$

where, $f_i(a^*) = \lambda^*$, if $a_i^* > 0$, and $f_i(a^*) = \lambda^*(C_{22})$ if $a_i^* = 0$. If the Perron-Frobenius eigenvector of C_{22} is not strictly positive, we can define r_i^* by iterating the argument above.

Conjecture 6 *For generic initial conditions (a, r) such that a is in the relative interior of K, and $r > 0$, the dynamics of (10)-(11) converges to the fixed point (a^*, r^*) defined by the proposition above. On the generic assumption that λ^* has multiplicity 1, (a^*, r^*) is the unique stable attractor of equations (10)-(11).*

The above conjecture is abundantly illustrated in the companion paper [3], where (a^*, r^*) is proved to be locally stable in low dimensional cases and is conjectured to preserve its local-stability properties in higher dimensions.

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